

## 論理回路[3SJ] 05 論理式の簡単化[1]

### [単語]

- 非標準形論理式

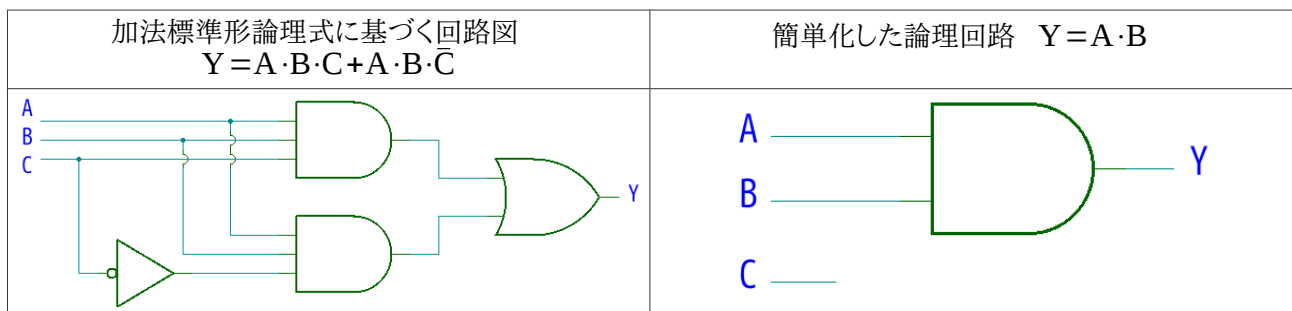
### [論理式を簡単化したい]

加法標準形・乗法標準形で記述した論理式は長い。  
同じ動作をする論理回路なら部品数は少ないほうがよい。

- 加法標準形論理式  $Y = A \cdot B \cdot C + A \cdot B \cdot \bar{C}$
- ブール代数の基本定理を使い、論理式は簡単化できる

$$Y = A \cdot B \cdot C + A \cdot B \cdot \bar{C}$$

=



### [論理式を簡単化するには?]

- ブール代数の基本定理を使い、論理式を変形・整理する
- カルノー図を使う

### [カルノー図(2変数)]

最小項とセルの対応

| $Y = A$   |   |   |   | $Y = \bar{A}$ |   |   |   | $Y = B$   |   |   |   | $Y = \bar{B}$ |   |   |   |
|-----------|---|---|---|---------------|---|---|---|-----------|---|---|---|---------------|---|---|---|
| 出力<br>[Y] |   | B |   | 出力<br>[Y]     |   | B |   | 出力<br>[Y] |   | B |   | 出力<br>[Y]     |   | B |   |
|           |   | 0 | 1 |               |   | 0 | 1 |           |   | 0 | 1 |               |   | 0 | 1 |
| A         | 0 |   |   | A             | 0 |   |   | A         | 0 |   |   | A             | 0 |   |   |
|           | 1 |   |   |               | 1 |   |   |           | 1 |   |   |               | 1 |   |   |

### [演習]

| $Y = A B$ |   |   |   | $Y = A + B$ |   |   |   | $Y = A \bar{B}$ |   |   |   | $Y = \bar{A} + B$ |   |   |   | $Y = \bar{A} \bar{B}$ |   |   |   |
|-----------|---|---|---|-------------|---|---|---|-----------------|---|---|---|-------------------|---|---|---|-----------------------|---|---|---|
| 出力<br>[Y] |   | B |   | 出力<br>[Y]   |   | B |   | 出力<br>[Y]       |   | B |   | 出力<br>[Y]         |   | B |   | 出力<br>[Y]             |   | B |   |
|           |   | 0 | 1 |             |   | 0 | 1 |                 |   | 0 | 1 |                   |   | 0 | 1 |                       |   | 0 | 1 |
| A         | 0 |   |   | A           | 0 |   |   | A               | 0 |   |   | A                 | 0 |   |   | A                     | 0 |   |   |
|           | 1 |   |   |             | 1 |   |   |                 | 1 |   |   |                   | 1 |   |   |                       | 1 |   |   |

| $Y = \bar{A} \bar{B}$ |   |   |   | $Y = \bar{A} + \bar{B}$ |   |   |   | $Y = \bar{A} + B$ |   |   |   | $Y = A B + \bar{A} \bar{B}$ |   |   |   | $Y = A \oplus B$ |   |   |   |
|-----------------------|---|---|---|-------------------------|---|---|---|-------------------|---|---|---|-----------------------------|---|---|---|------------------|---|---|---|
| 出力<br>[Y]             |   | B |   | 出力<br>[Y]               |   | B |   | 出力<br>[Y]         |   | B |   | 出力<br>[Y]                   |   | B |   | 出力<br>[Y]        |   | B |   |
|                       |   | 0 | 1 |                         |   | 0 | 1 |                   |   | 0 | 1 |                             |   | 0 | 1 |                  |   | 0 | 1 |
| A                     | 0 |   |   | A                       | 0 |   |   | A                 | 0 |   |   | A                           | 0 |   |   | A                | 0 |   |   |
|                       | 1 |   |   |                         | 1 |   |   |                   | 1 |   |   |                             | 1 |   |   |                  | 1 |   |   |

論理回路[3SJ] 05 論理式の簡単化[1]

[グレイコード]

2ビット

00,01,11,10

3ビット

000,001,011,010,110,111,101,100

4ビット

0000,0001,0011,0010,0110,0111,0101,0100,  
1100,1101,1111,1110,1010,1011,1001,1000

[カルノー図(3変数)]

| 出力<br>[Y] |   | B C |    |    |    |
|-----------|---|-----|----|----|----|
|           |   | 00  | 01 | 11 | 10 |
| A         | 0 |     |    |    |    |
|           | 1 |     |    |    |    |

| 出力<br>[Y] |    | C |   |
|-----------|----|---|---|
|           |    | 0 | 1 |
| AB        | 00 |   |   |
|           | 01 |   |   |
|           | 11 |   |   |
|           | 10 |   |   |

カルノー図は横長・縦長どちらでもよい

最小項

| 出力<br>[Y] |   | B C |    |    |    |
|-----------|---|-----|----|----|----|
|           |   | 00  | 01 | 11 | 10 |
| A         | 0 |     |    |    |    |
|           | 1 |     |    |    |    |

Y=

| 出力<br>[Y] |   | B C |    |    |    |
|-----------|---|-----|----|----|----|
|           |   | 00  | 01 | 11 | 10 |
| A         | 0 |     |    |    |    |
|           | 1 |     |    |    |    |

最小項

| 出力<br>[Y] |    | C |   |
|-----------|----|---|---|
|           |    | 0 | 1 |
| AB        | 00 |   |   |
|           | 01 |   |   |
|           | 11 |   |   |
|           | 10 |   |   |

Y=

| 出力<br>[Y] |    | C |   |
|-----------|----|---|---|
|           |    | 0 | 1 |
| AB        | 00 |   |   |
|           | 01 |   |   |
|           | 11 |   |   |
|           | 10 |   |   |

[カルノー図(4変数)]

最小項

| 出力<br>[Y] |    | C D |    |    |    |
|-----------|----|-----|----|----|----|
|           |    | 00  | 01 | 11 | 10 |
| AB        | 00 |     |    |    |    |
|           | 01 |     |    |    |    |
|           | 11 |     |    |    |    |
|           | 10 |     |    |    |    |

Y=

| 出力<br>[Y] |    | C D |    |    |    |
|-----------|----|-----|----|----|----|
|           |    | 00  | 01 | 11 | 10 |
| AB        | 00 |     |    |    |    |
|           | 01 |     |    |    |    |
|           | 11 |     |    |    |    |
|           | 10 |     |    |    |    |

論理回路[3SJ] 05 論理式の簡単化[1]

[隣接する"1"に着目してグループ化]

| 出力<br>[Y] |    | C D |    |    |    |
|-----------|----|-----|----|----|----|
|           |    | 00  | 01 | 11 | 10 |
| AB        | 00 | 1   | 1  |    |    |
|           | 01 |     |    |    |    |
|           | 11 |     |    |    |    |
|           | 10 |     |    |    |    |

| 出力<br>[Y] |    | C D |    |    |    |
|-----------|----|-----|----|----|----|
|           |    | 00  | 01 | 11 | 10 |
| AB        | 00 | 1   |    |    |    |
|           | 01 | 1   |    |    |    |
|           | 11 |     |    |    |    |
|           | 10 |     |    |    |    |

| 出力<br>[Y] |    | C D |    |    |    |
|-----------|----|-----|----|----|----|
|           |    | 00  | 01 | 11 | 10 |
| AB        | 00 | 1   |    |    |    |
|           | 01 | 1   |    |    | 1  |
|           | 11 |     |    |    |    |
|           | 10 | 1   |    |    |    |

| 出力<br>[Y] |    | C D |    |    |    |
|-----------|----|-----|----|----|----|
|           |    | 00  | 01 | 11 | 10 |
| AB        | 00 |     |    | 1  |    |
|           | 01 |     | 1  | 1  | 1  |
|           | 11 |     |    |    |    |
|           | 10 |     |    |    |    |

| 出力<br>[Y] |    | C D |    |    |    |
|-----------|----|-----|----|----|----|
|           |    | 00  | 01 | 11 | 10 |
| AB        | 00 | 1   |    |    | 1  |
|           | 01 |     |    |    |    |
|           | 11 |     |    |    |    |
|           | 10 | 1   |    |    | 1  |

| 出力<br>[Y] |    | C D |    |    |    |
|-----------|----|-----|----|----|----|
|           |    | 00  | 01 | 11 | 10 |
| AB        | 00 | 1   | 1  |    |    |
|           | 01 | 1   | 1  |    |    |
|           | 11 | 1   | 1  |    |    |
|           | 10 | 1   | 1  |    |    |

[グループ化のルール]

論理回路[3SJ] 05 論理式の簡単化[1]

[練習問題](p39) 【5】つぎの論理式を簡単化しなさい。

(1)  $Y =$

| 出力<br>[Y] |   | B |   |
|-----------|---|---|---|
|           |   | 0 | 1 |
| A         | 0 |   |   |
|           | 1 |   |   |

(2)  $Y =$

| 出力<br>[Y] |   | B C |    |    |    |
|-----------|---|-----|----|----|----|
|           |   | 00  | 01 | 11 | 10 |
| A         | 0 |     |    |    |    |
|           | 1 |     |    |    |    |

(3)  $Y =$

| 出力<br>[Y] |   | B C |    |    |    |
|-----------|---|-----|----|----|----|
|           |   | 00  | 01 | 11 | 10 |
| A         | 0 |     |    |    |    |
|           | 1 |     |    |    |    |

(4)  $Y =$

| 出力<br>[Y] |    | C D |    |    |    |
|-----------|----|-----|----|----|----|
|           |    | 00  | 01 | 11 | 10 |
| AB        | 00 | 1   | 1  |    |    |
|           | 01 |     |    |    |    |
|           | 11 |     |    |    |    |
|           | 10 |     |    |    |    |

(5)  $Y =$

| 出力<br>[Y] |    | C D |    |    |    |
|-----------|----|-----|----|----|----|
|           |    | 00  | 01 | 11 | 10 |
| AB        | 00 |     |    |    |    |
|           | 01 |     |    |    |    |
|           | 11 |     |    |    |    |
|           | 10 |     |    |    |    |

(6)  $Y =$

| 出力<br>[Y] |    | C D |    |    |    |
|-----------|----|-----|----|----|----|
|           |    | 00  | 01 | 11 | 10 |
| AB        | 00 |     |    |    |    |
|           | 01 |     |    |    |    |
|           | 11 |     |    |    |    |
|           | 10 |     |    |    |    |

[演習]つぎの真理値表から論理式を求めなさい。論理式は簡単化しなさい。

(1)

| 入力 |   |   | 出力 |
|----|---|---|----|
| A  | B | C | f  |
| 0  | 0 | 0 | 1  |
| 0  | 0 | 1 | 0  |
| 0  | 1 | 0 | 1  |
| 0  | 1 | 1 | 1  |
| 1  | 0 | 0 | 0  |
| 1  | 0 | 1 | 0  |
| 1  | 1 | 0 | 1  |
| 1  | 1 | 1 | 1  |

(2)

| 入力 |   |   |   | 出力 |
|----|---|---|---|----|
| A  | B | C | D | f  |
| 0  | 0 | 0 | 0 | 0  |
| 0  | 0 | 0 | 1 | 0  |
| 0  | 0 | 1 | 0 | 1  |
| 0  | 0 | 1 | 1 | 0  |
| 0  | 1 | 0 | 0 | 1  |
| 0  | 1 | 0 | 1 | 0  |
| 0  | 1 | 1 | 0 | 0  |
| 0  | 1 | 1 | 1 | 1  |
| 1  | 0 | 0 | 0 | 1  |
| 1  | 0 | 0 | 1 | 0  |
| 1  | 0 | 1 | 0 | 0  |
| 1  | 0 | 1 | 1 | 1  |
| 1  | 1 | 0 | 0 | 0  |
| 1  | 1 | 0 | 1 | 0  |
| 1  | 1 | 1 | 0 | 1  |
| 1  | 1 | 1 | 1 | 1  |